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MODELING THE FULL MIMO MATRIX USING THE RICHNESS FUNCTION

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ABSTRACT

A richness function depending only on normalized eigenvalues of the MIMO channel matrix is introduced. The function is a good measure for comparing environments illuminated by the same antennas. A new channel model incorporating the effective number of scatterers and propagation between the antenna environments s introduced, and the richness function is introduced to extract channel parameters. Examples of indoor environments consisting of offices on the same and on different floors are given.

1. INTRODUCTION

In a MIMO situation we are interested in the capacity and creating channel models which mimic the real world as closely as possible. The capacity and its statistical variation depends on many factors, such as the distribution of energy in the environment, the antennas and their arrangement, and on the path loss, or rather the *SNR*. Most previous efforts of modelling the MIMO channel have been concerned with the effect of correlated antennas, which reduces the diversity and the channel capacity [1, 2, 3]. This reflects what can be 'seen' from the antenna positions, but there may be phenomena in between, which may be important. This is the main subject of this paper. In order to compare different environments in detail we introduce the richness function [4,5], which depends only on the eigenvalues, and not on the signal-to-noise ratio. The richness depends then only on the antennas and the environment, and by using the same antennas in different environments, we are able to analyse the difference between environments in great detail. This is not to suggest that the path loss is not important; on the contrary, it has a major impact on the capacity, but it is of advantage to separate the effects.

A simple random coupling matrix is introduced to describe the 'black box' between the two environments close to the antennas. It is inspired by the Gesbert model [6], which was originally developed for the outdoor scenario. The matrix can model the continuous change from a full rank system to a keyhole as the two extremes.

An additional parameter introduced is the number of effective scatterers. It is well known that if the number of paths is less than the number of antennas, the rank is reduced, but a number larger than this has also an effect of reducing capacity or richness.

Various indoor environments have been measured with a multi channel sounder at 5.8 GHz [7], and the parameters are extracted by minimizing the difference between experimentally measured richness and channel model richness.

The paper is organized in the following way. In section two the richness function is introduced, followed by a development of the MIMO channel model. Model parameters are then extracted from measurements and discussed.

2. THE MULTIPATH RICHNESS

The capacity or spectral efficiency of an environment depends on the antenna structures at each end, knowledge of channel information at the ends, correlations between antennas and between paths, the distribution of scatterers and most importantly on the SNR level. It is most easily expressed in the well-known formula for capacity [8]

$$C = \log_2(\det(\boldsymbol{I} + \frac{P}{M}\boldsymbol{HH}^H)) \quad b/s/Hz \qquad (1)$$

which may be calculated for many different cases once the channel matrix $\boldsymbol{H}$ has been measured. P is the power normalised to the noise power, the *SNR*, and the number of transmit antennas is M. It is of interest to find a simple relationship or curve, which expresses the multipath richness of the channel matrix without reference to the power or *SNR*. Of course, one could use the capacity for a given *SNR* as such a measure, but that is just one number, which does not contain any additional information. As suggested in [9] the *EDOF* (effective degrees of freedom) is essentially the slope of capacity versus *SNR* at one value of *SNR* and gives an indication of the rank of the system. In [10] the relative sum of the channel singular values of the channel matrix is used. For the moment we assume N receive antennas, where $N<M$, which means that the maximum number of non-zero eigenvalues equals N. The number and magnitude of significant eigenvalues

or singular values determines this richness, and in this note we first explore this by letting *SNR* become so large that we can ignore the identity matrix $\boldsymbol{I}$ in (A1.1). As it has been emphasised in [11] we can think of the eigenvalues of ***HH'*** as gains of the independent, orthogonal channels. Expressing the determinant through the eigenvalues we can obtain a convenient measure for the multipath richness, independent of *SNR*, and if expressed in dB also a convenient measure of the gains. The channel matrices are normalised to have mean gain of 1 (0 dB), which means that there is a constraint on the total gain,

$$\sum_{1}^{N} \lambda_i = NM \tag{2}$$

Using the arguments above to expand equation (1) we find

$$\begin{aligned} C &= \log_2(\det(\boldsymbol{I} + \frac{P}{M}\boldsymbol{HH}')) \\ &= \log_2(\prod_{i=1}^{N}(1+\frac{P}{M}\lambda_i)) \\ &\approx \log_2(\prod_{i=1}^{N}(\frac{P}{M}\lambda_i)) \quad \mathit{for}\ \frac{P}{M}\lambda_i > 1 \\ &= \boldsymbol{N}\log_2(\frac{P}{M}) + \sum_{i=1}^{N}\log_2(\lambda_i) \\ &= 0.33\boldsymbol{N}(\frac{P}{M}(dB)) + \sum_{i=1}^{N}\log_2(\lambda_i) \end{aligned} \tag{3}$$

where the factor 0.33 stems from the transformation from log_2 to dB. The richness curve (or vector) is now defined as the cumulative sum of the log of the eigenvalues

$$R(k) = \sum_{i=1}^{k} log_2(\lambda_i) \quad k = 1, N \tag{4}$$

All the *N* numbers are part of the definition of the richness vector. Occasionally we will use the term richness for *R(N)*.

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As will be seen this measure has a significant amount of information concerning the multipath richness, and apart from an easily calculated constant term depending on the *SNR* the capacity equals the richness. It is the same richness no matter which end is the transmitter. The eigenvalues are ordered in decreasing order.

The final richness (and capacity) using all eigenvalues and sufficiently high power equals *R(N)*, but if

$$\frac{P}{M}\lambda_t < 1 \tag{5}$$

for a particular value of *i=t* then the eigenvalues from *t* and above do not contribute to the capacity, and we can use *R(t)* as the measure of richness for that particular value of *SNR*.

If the eigenvalues are known at the transmitter we can use water filling, in which case the capacity equals the richness plus a term depending on the power without any approximations.

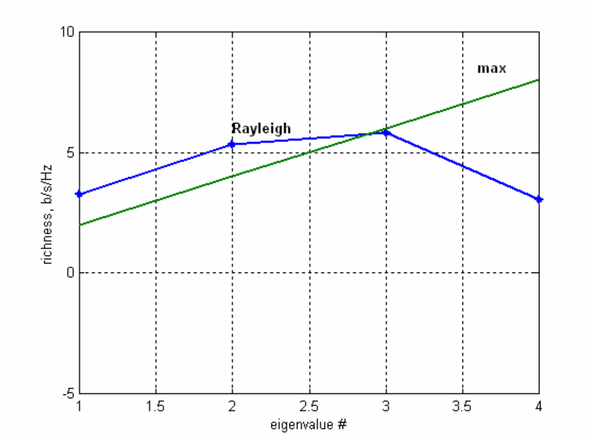


Figure 1 Mean richness curves for (N,M)=(4,4). Ideal Rayleigh case. Maximum value is 8, obtained in the case when all eigenvalues are equal.

A simple case of mean richness for a (4,4) MIMO is shown in Figure 1 for an ideal Rayleigh with independent paths. Also shown is the theoretical maximum of 8 b/s/Hz, obtained when all eigenvalues are equal.

3. CHANNEL MODEL

The Gesbert model [6] may be written as

$$\boldsymbol{H} = \boldsymbol{\Phi_R G_R X_{RT} G_T \Phi_T}^{\boldsymbol{T}} \tag{6}$$

where the *G*-matrices contain N_s random, complex Gaussian fading scatterers, and $\boldsymbol{X_{RT}}$ contain only random phase terms, with a uniform distribution of phase between zero and $2\pi\ \delta\theta$. The Φ-matrices are the usual square root correlation matrices. The black box separating the two layers is now described by two parameters, N_s and $\delta\theta$. It is assumed that N_s is the same for both layers, which is not necessarily the case. A sketch of the model is shown in Figure 2. In the original Gesbert model the paths connecting the scatterers would be real paths, but here we can interpret it more loosely as a common environment shared by the paths to a smaller or larger degree.

When N_s is significantly less than *NM* the richness decreases (apparently because the number of scatterers is not sufficient for the independence of the *NM* channel coefficients). It has also an important effect on the outage capacity.

The $\delta\theta$ – factor expresses the limited excursion of the propagation paths, when they share a common general path. For $\delta\theta = 2\pi$ the situation is similar (although not identical) to a single bounce situation, and in the other extreme $\delta\theta = 0$ we have the keyhole effect with only one eigenvalue.

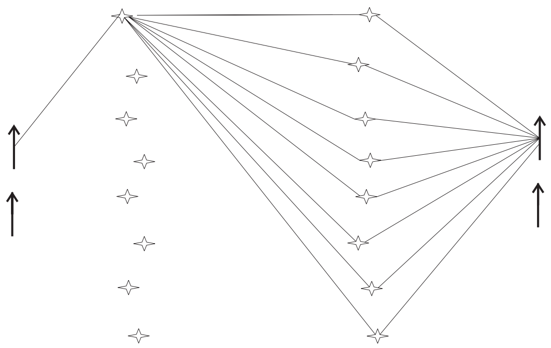


Figure 2. The two-layer model where each scatterer in the first layer is radiating to each scatterer in the second layer.

The Φ-matrices may be determined in the usual way by either describing the angular distribution of energy, like clusters and their spreads, or experimentally by determining the correlations between the antennas at both ends. It is here important to stress that a sufficient number of independent measurements are needed for a statistically significant correlation matrix. In principle movement of the antennas locally is the best method, supplemented by frequency diversity, if the frequency coherence bandwidth is sufficient small. Once the correlations have been determined, the new parameters N_s and $\delta\theta$ are found by matching the mean richness from experiment to model, minimizing the rms error. Before studying real environments, let us first simulate the effect of the new parameters on the richness.

4. SIMULATION OF EFFECT OF COUPLING MATRIX

4a Number of scatterers

Normally, in an indoor environment the number of scatterers will be large compared with *NM,* especially for small arrays. Nevertheless, let us use the *(4, 4)* as an example. In Figure 3 is shown the mean richness degradation [5] for N_s= 2,4,8,12,16, and 20.

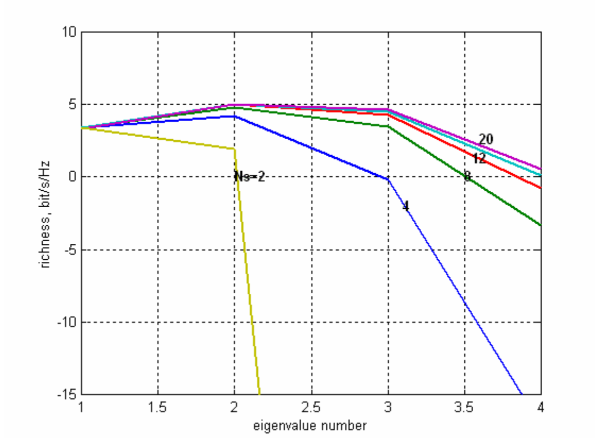


Figure 3 Mean richness for two (4, 4) linear arrays with λ/2 spacing in a wide scattering (low correlation) environment as a function of the number of effective scatterers.

It is clear that for N_s =2 there are only two eigenvalues, while we have a full rank matrix for all the other cases, although with much reduced richness for the smaller numbers. The reason why the asymptotic value for large N_s differs from the Rayleigh case in Figure 1 is a small, but finite remaining correlation.

4b Phase fluctuations of coupling matrix.

The $\boldsymbol{X}_{RT}$ matrix in (6) describes the connection between the two layers and is here given as a pure phase fluctuation where the phase is uniformly distributed between 0 and $2\pi\ \delta\theta$. Simulations for the (4,4) case are shown in Figure 4.

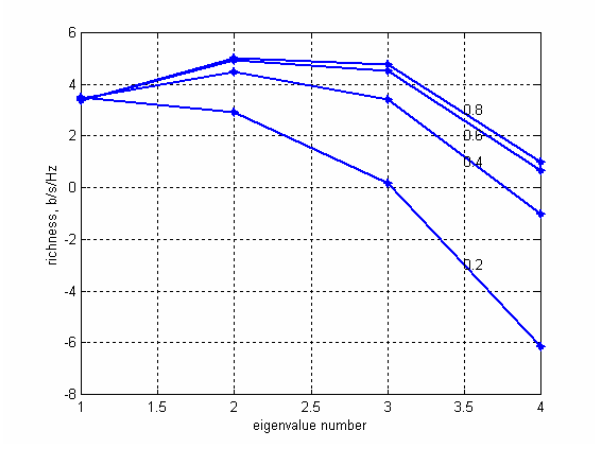


Figure 4. Mean richness for two (4, 4) linear arrays with λ/2 spacing in a wide scattering (low correlation) environment as a function of the phase fluctuations of the coupling matrix.

Values of $\delta\theta$ larger than *0.6* do not have a significant effect, while smaller values diminish the richness and thus the capacity for this size of arrays.

5. MEASUREMENTS

The measured data used in this work are obtained using a 16x32 MIMO channel sounder based on the correlation principle. The sounding bandwidth is about 100MHz which is obtained by transmitting the PN sequence at a chip rate of 100MHz using BPSK. The carrier frequency is 5.8GHz.

Each of the 16 transmitters simultaneously output a PN sequence using a 1W power amplifier. The system is designed to be flexible so that any set of PN sequences can be used, as required by the type of measurements. In the current work an m-sequence of length 1023 is used for all the transmitters, where each transmitter has a unique code offset so that the different channels can be separated in the receiver.

All frequency sources in both the transmitter and receiver are phase locked to rubidium frequency standards, so that complex impulse responses can be measured.

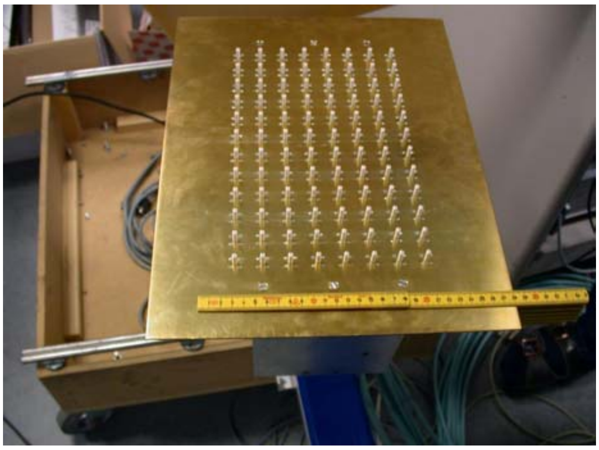


Figure 5 Planar array of monopoles at receiver

The receiver has four parallel branches each with a separate AGC circuit and sampler. Four channels are measured truly in parallel and via switching this is repeated 8 times to obtain the 32 receive channels. For the current setup with a 1023 chip PN sequence, measurement of the 32 channels takes about 572 µs, using four times averaging of each measurement in order to improve the dynamic range. Assuming a maximum speed of 1m/s of a receiver or transmitter moving in an indoor environment, the total measurement time corresponds to about $\lambda/90$, where λ is the wavelength. The measurements part of the current work the full MIMO channel is measured in a periodic manner at a rate of 60Hz, or about every 17ms.

The received signals are sampled at an IF stage and in a post-processing procedure the complex impulse responses are obtained, which are then compensated for the system response using back-to-back measurements.

The measurements discussed subsequently were obtained with a planar array of monopole antennas at the receiver arranged in a 8x12 rectangular grid where the two outermost rows on all sides are dummy elements so that the array effectively is 4x8. The monopole antennas are about 0.3 λ in length and spaced about 0.5 λ apart. Figure 1 shows the receiver antenna. The transmitter array is similar in construction but instead is a 8x8 grid where the active elements are in a 4x4 grid.

The receiver array is mounted on a sledge controlled by a step motor so that the array can be moved in a linear fashion during the measurements. The height of the array is 94cm above the floor. In the current measurement campaign the movement of the
the sledge from one end to the other was set to 1m, which takes 30s. Given the above mentioned measurement rate of 60Hz, a single measurement run consists of 1800 samples of the 16x32 complex impulse responses.

Measurement Scenarios

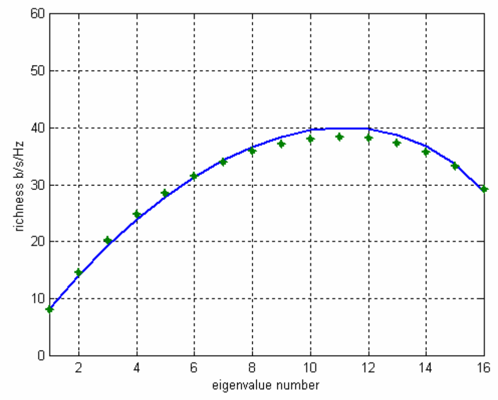
All the measurements were made within the same modern four story (including basement) office building. The building is primarily made of reinforced concrete with an outer brick wall and with most inner partitions made in light plaster board construction. The floors/ceilings of each level are also made of concrete. Numerous measurements were included in the campaign. For the measurements discussed in the current work the Tx array remained fixed during the individual measurement while the receiver moved on the sledge, as described above.

The following widely different scenarios have been selected. The scenarios are chosen to mimic a point-to-point communications link between two terminals or a base station to mobile kind of communication link.

Two different levels

For the remaining measurements the Tx antenna array was positioned on a table with the monopoles vertically oriented and the ground plane at a height of 88cm above the floor.

For the level crossing measurements the Tx array is on the 1st floor and the Rx array is on the 2nd floor. In this situation most of the energy can be expected to propagate via corridors and staircases. The two floors are connected to a main entrance hall which covers the full height of the building.



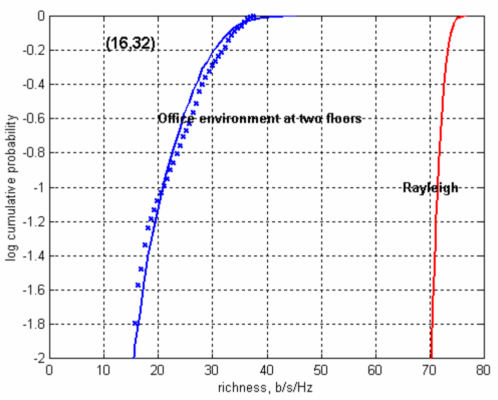
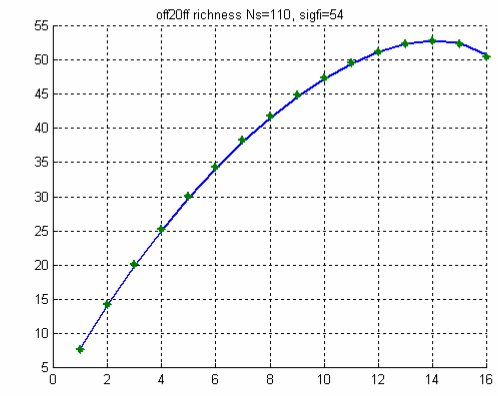


Figure 6 a) and b). a) shows the experimental mean values of richness curve with fitting to theory Ns=72 and δθ=0.8. b) shows the distribution of richness R(N).

The agreement for the distribution of richness is very good, since it does not require any additional conditions, just the fitting of the mean values in fig 6a. Thus we can be sure that both the mean capacity and outrage capacity will agree well for all values of *SNR*.

Office to Office Measurements

In this scenario both the Tx and Rx arrays are located inside small offices next to the 2nd floor corridor. The following two measurements are included in this work:



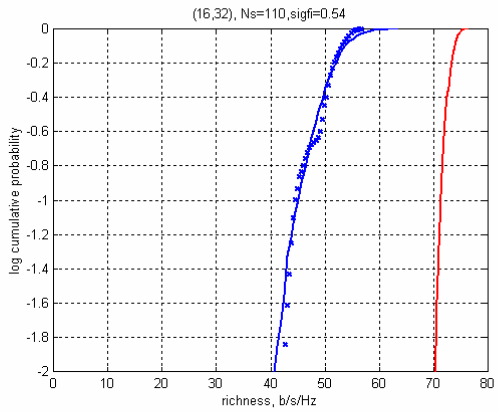


Figure7 a) and b). a) shows the experimental mean values of richness curve with fitting to theory, Ns=110, and δθ=0.54. b) shows the distribution of richness R(N).

The richness is now much larger in this case, partly due to the larger number of effective scatterers. Again there is a very good fit to the distribution of richness, although only the mean values have been used for finding the optimum parameters.

6. CONCLUSION

The richness curve (or form factor) seems to be a sensitive measure of the potential capacity in various environments, showing that the Gesbert channel model also may be used for indoor environments. The model is fitted to the mean values of richness, while the distribution function follows automatically without the need for further assumptions. The agreement with measurements is gratifying, although more measurements are needed to separate with greater accuracy the influence of correlations near the antennas from the effect of the in-between environments.

Acknowledgement

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